The 10th East Asian Conference in Harmonic Analysis and Applications

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Talks Information

(1) **Kohei Akase** (Osaka University, u306026b@ecs.osaka-u.ac.jp)
Title: Well-posedness for a nonlinear Schrödinger equation with quadratic derivative nonlinearity.
Abstract: In this talk, we consider the Cauchy problem of a nonlinear Schrödinger equation that has nonlinearities $\partial_x(u^2)$ and $\partial_x(|u|^2)$. Christ (2003, preprint) proved that the Cauchy problem is ill-posed in any L^2-based Sobolev spaces when the nonlinearity is $\partial_x(u^2)$. On the other hand, Ozawa (1998) showed well-posedness when imposing boundedness of primitive of initial data. We consider the well-posedness with boundedness of primitive. Moreover, we prove the well-posedness in Sobolev spaces when considering that the coefficients of two nonlinearities have some relationship.

(2) **Junsik Bae** (Ulsan National Institute of Science and Technology, junsikbae@unist.ac.kr)
Title: Nonexistence of multi-dimensional solitary waves in unmagnetized plasma.
Abstract: We study the nonexistence of multi-dimensional solitary waves for the Euler-Poisson system governing ion dynamics. It is well-known that the one-dimensional Euler-Poisson system has solitary waves travel faster than the ion-sound speed. In contrast, we show that the two-dimensional and three-dimensional models do not admit nontrivial irrotational spatially localized traveling waves. We derive some Pohozaev type identities associated with the energy and density integrals.

(3) **Lu Chen** (Beijing Institute of Technology, chenlu5818804@163.com)
Title: Uniqueness and quantization analysis for positive solutions of Trudinger-Moser equation.
Abstract: In this talk, I will first introduce our uniqueness result for positive solutions of Trudinger-Moser equation in unit ball of Euclidean space or Hyperbolic space, this result can be seen as an important step for the uniqueness of maximizers of Trudinger-Moser inequalities. Based on this uniqueness result, we will develop a new strategy to establish the quantization property of elliptic equations with the critical exponential growth

in the balls of hyperbolic spaces, and obtain the multiplicity and non-existence of positive critical points for super-critical Trudinger-Moser functional. Our method for the quantization property and non-existence of the critical points avoids using the complicated blow-up analysis used in the literature. This is a joint work with Prof. Lu from Connecticut University and Prof. Zhu from Jiangsu Univerity.

(4) **Yanping Chen** (University of Science and Technology Beijing, yanpingch@ustb.edu.cn)
Title: Some progress on variational operators.
Abstract: In this talk, we proved some variational inequalities of the singular integral operators.

(5) **Xiumin Du** (Northwestern University, xdu@northwestern.edu)
Title: Falconer distance set problem.
Abstract: A classical question in geometric measure theory, introduced by Falconer in the early 80s is, how large does the Hausdorff dimension of a compact subset of $\mathbb{R}^d$ need to be to ensure that the Lebesgue measure of the set of its pairwise Euclidean distances is positive. In this talk, I'll report some new progress on this conjecture. The talk is based on an ongoing project joint with Yumeng Ou, Kevin Ren and Ruixiang Zhang.

(6) **Zunwei Fu** (Linyi University, fuzunwei@lyu.edu.cn)
Title: Riesz transform associated with the fractional Fourier transform and applications in image edge detection.
Abstract: Since Zayed [Zayed, 1998] introduced the fractional Hilbert transform related to the fractional Fourier transform, this transform has been widely concerned and applied in the field of signal processing. Recently, Chen, the first, second and fourth authors [Chen et al, 2021] attribute it to the operator corresponding to fractional multiplier, but it is only limited to 1-dimensional case. This paper naturally considers the high-dimensional situation. We introduce the fractional Riesz transform associated with fractional Fourier transform, in which the chirp function is the key factor and the technical barriers to be overcome. Furthermore, after equipping with chirp functions, we introduce and investigate the boundedness of singular integral operators, the dual properties of Hardy spaces and BMO spaces as well as the applications of theory of fractional multiplier in partial differential equation, which completely matched some classical results. Through numerical simulation, we give the physical and geometric interpretation of the higher dimensional fractional multiplier theorem. Finally, we present the application of fractional Riesz transform in edge detection, which verifies the prediction proposed in [Xu et al,2016]. Moreover, the application presented in this paper can also be considered as the high-dimensional case of the application of the continuous fractional Hilbert transform in edge

detection in [Pei and Yeh, 2000]. This is joint work with Prof. Grafakos, Prof. Lin, Prof. Wu and Dr. Yang.

(7) **Denny I. Hakim** (Bandung Institute of Technology, Indonesia, dennyivanalhakim@gmail.com)
Title: Some endpoint estimates of Olsen's type inequalities and commutators of fractional operators in Lebesgue spaces.
Abstract: In this talk, we will discuss some recents results about some endpoint estimates of the fractional integral operators and the fractional maximal operators in Lebesgue spaces. In particular, we apply the result of Spector about L1-estimates of Riesz potentials to prove some endpoint estimates for fractional maximal operators in Lebesgue spaces. We also prove the corresponding estimates for Olsen's inequalities and commutators for both fractional integral operators and fractional maximal operators. These results are joint work with Wono Setya Budhi, Daniel Salim, Muhamad Jamaludin, and Verrel Rievaldo Wijaya.

(8) **Seheon Ham** (Seoul National University, seheonham@snu.ac.kr)
Title: Remarks on dimension of union of curves.
Abstract: We study an analogue of Marstrand's circle packing problem for curves in higher dimensions. We consider collections of curves which are generated by translation and dilation of a curve γ in $\mathbb{R}^d$, i.e., $x + t\gamma$, $(x,t) \in \mathbb{R}^d \times (0,\infty)$. For a Borel set $F \subset \mathbb{R}^d \times (0,\infty)$, we show the unions of curves $\bigcup_{(x,t)\in F}(x+t\gamma)$ has Hausdorff dimension at least $\alpha+1$ whenever F has Hausdorff dimension bigger than $\alpha \in (0, d-1)$. We also obtain results for unions of curves generated by multi-parameter dilation of γ. One of the main ingredients is a local smoothing type estimate (for averages over curves) relative to fractal measures.

This talk is bases on recent work with Herym Ko, Sanghyuk Lee, and Sewook Oh.

(9) **Sunggeum Hong**, (Chosun University, skhong@chosun.ac.kr)
Title: Hörmander type multiplier theorem for multilinear pseudo-differential operators.
Abstract: In this talk we establish a Hörmander type theorem for multilinear pseudo-differential operators, which is also a generalization of the results in [1] to symbols depending on the spatial variable. Most known results for multilinear pseudo-differential operators were obtained by assuming their symbols satisfy pointwise derivative estimates(Mihlin-type condition), that is, their symbols belong to some symbol classes n-$\mathcal{S}^m_{\rho,\delta}(\mathbb{R}^d)$, $0 \le \delta \le \rho \le 1$, $0 \le \delta < 1$ for some $m \le 0$. However, we shall consider multilinear pseudo-differential operators whose symbols have limited smoothness described in terms of function space and not in a pointwise form (Hörmander type condition). Our conditions for symbols are weaker

than the Mihlin-type condition in two senses: the one is that we only assume that the first-order derivative conditions in the spatial variable and lower-order derivative conditions in the frequency variable, and the other is that we make use of L^2-average condition rather than pointwise derivative conditions for the symbols. As an application, we obtain some mapping properties for the multilinear pseudo-differential operators associated with symbols belonging to the classes n-$\mathcal{S}^m_{\rho,\delta}(\mathbb{R}^d)$, $0 \le \rho \le 1$, $0 \le \delta < 1$ and $m \le 0$. Moreover, it can be pointed out that our results are applied to wider classes of symbols which do not belong to the traditional classes of symbols n-$\mathcal{S}^m_{\rho,\delta}(\mathbb{R}^d)$. This is a joint work with Yaryong Heo and Chan Woo Yang.

Reference: [1] Jongho Lee, Yaryong Heo, Sunggeum Hong, Jin Bong Lee, Bae Jun Park, Yejune Park, and Chan Woo Yang, *The Hörmander multiplier theorem for n-linear operators*, Math. Ann. **381** (2021), no. 1-2, 499-555.

(10) **Minbeom Kang** (Yonsei University, minbeom_k@yonsei.ac.kr)
Title: Newton Polygons and Oscillatory Integral Operators.
Abstract: In this talk, we discuss whether the norm of the oscillatory integral operator on a global domain decays, and if it decays, then how the rate of this decay can be determined from the properties of the phase function. In particular, for this purpose, we introduce some geometric properties of the Newton polygon. This talk is based on joint work with Joonil Kim.

(11) **Takeshi Kawazoe** (Keio University, kawazoe@sfc.keio.ac.jp)
Title: A real Hardy space on the Jacobi hypergroup and its atom and quark decompositions.
Abstract: Let $(\mathbb{R}_+, \Delta, *)$ be the Jacobi hypergroup. The Hardy space $H^1(\Delta)$ is defined as a collection of f in $L^1_{\rm loc}(\Delta)$ such that the radial maximal function $M_\psi f = \sup_{t>0} |f * \psi_t|$ belongs to $L^1(\Delta)$, where ψ is a suitable compactly supported even function on $\mathbb{R}$ and the dilation ψ_t of ψ is given by $\psi_t(x) = \frac{1}{t\Delta(x)}\Delta(\frac{x}{t})\psi(\frac{x}{t})$. In a series of papers a function f in $H^1(\Delta)$ is characterized in terms of its Abel transform $\mathcal{A}(f)$. In this talk we give an atomic decomposition of $H^1_0(\Delta)$ and a quark decomposition of each atom.

(12) **Jongchon Kim** (City University of Hong Kong, Jongckim@cityu.edu.hk)
Title: Nikodym sets and maximal functions associated with spheres.
Abstract: Any set containing a sphere centered at every point cannot have 0 Lebesgue measure. This is a consequence of the L^p boundedness of the spherical maximal function. On the other hand, there are sets of 0 Lebesgue measure which contain a large family of spheres, which may be considered as Kakeya/Nikodym sets for spheres. In this talk, I will introduce maximal

functions associated with these sets and discuss their L^p mapping properties.

(13) **Hyerim Ko** (Seoul National University, kohr@snu.ac.kr)
Title: Sobolev regularity of restricted X-ray transforms.
Abstract: This talk is concerned with regularity property of a restricted X-ray transform. We prove sharp Lp Sobolev regularity estimates for restricted X-ray transforms in all dimensions, which extends earlier result of PramanikSeeger in R3. This is achieved by constructing inductive argument and making use of the decoupling inequality for curves. This is joint work with Sanghyuk Lee and Sewook Oh.

(14) **Youngwoo Koh** (Kongju National University, ywkoh@kongju.ac.kr).
Title: Time splitting method for nonlinear Schrödinger equation with rough initial data in L^2.
Abstract: We establish convergence results related to the operator splitting scheme on the Cauchy problem for the nonlinear Schrödinger equation with rough initial data in L^2,

$$\begin{cases} i\partial_t u + \Delta u = \lambda |u|^p u, & (x,t) \in \mathbb{R}^d \times \mathbb{R}_+, \\ u(x,0) = \phi(x), & x \in \mathbb{R}^d, \end{cases}$$

where $\lambda \in \{-1, 1\}$ and $p > 0$. While the Lie approximation Z_L is known to converge to the solution u when the initial datum ϕ is sufficiently smooth, the convergence result for rough initial data is open to question. In this talk, for rough initial data $\phi \in L^2(\mathbb{R}^d)$, we prove the L^2 convergence of the filtered Lie approximation Z_{flt} to the solution u in the mass-subcritical range, $\max\left\{1, \frac{2}{d}\right\} \le p < \frac{4}{d}$. Furthermore, we provide a precise convergence result for radial initial data $\phi \in L^2(\mathbb{R}^d)$. This talk is based on the joint work with Hyung Jun Choi(Korea University of Technology and Education) and Seonghak Kim(Kyungpook National University).

(15) **Jin Bong Lee**, (Seoul National University, jinblee@snu.ac.kr).
Title: L^p estimates for lacunary maximal functions associated with certain multilinear averages.
Abstract: In this talk, we consider certain types of multilinear averages and associated lacunary maximal operators. To obtain multilinear estimates for the lacunary maximal operators, we make use of simple L^p improving and Sobolev regularity estimates of multilinear averages. Then by means of Calderón-Zygmund theory, we obtain L^p estimates of the lacunary maximal functions for some $0 < p < 1$. We also study that the L^p improving estimates of multilinear averages can be improved under specific conditions by making use of the nonlinear Brascamp-Lieb inequality.

(16) **Juyoung Lee** (Juyoung Lee, Seoul National University, ljy219@snu.ac.kr)
Title: The elliptic maximal function.
Abstract: We study the elliptic maximal functions defined by averages over ellipses and rotated ellipses which are multi-parametric variants of the circular maximal function. We prove that those maximal functions are bounded on L^p for some $p \neq \infty$. For this purpose, we obtain some sharp multi-parameter local smoothing estimates.

(17) **Sanghyuk Lee** (Seoul National University, shklee@snu.ac.kr)
Title: Endpoint bounds on the Hermite spectral projection.
Abstract: This talk concerns L^2-L^q bounds on the Hermite spectral projection operator Π_λ in $\mathbb{R}^d$. For $d \geq 2$, the optimal L^2-$L^{2(d+3)/(d+1)}$ bound on Π_λ has been left unsettled for a long time. We prove this missing endpoint case for every $d \geq 3$. Our result is based on a new phenomenon: improvement of bounds due to asymmetric localization near the sphere.

(18) **Wenjuan Li** (Northwestern Polytechnical University, liwj@nwpu.edu.cn)
Title: On L^p-improving bounds for maximal operators associated with curves of finite type in the plane.
Abstract: In this paper, we study the L^p-improving property for the maximal operators along a large class of curves of finite type in the plane with dilation set $E \subset [1, 2]$. The L^p-improving region depends on the order of finite type and the fractal dimension of E. In particular, various impacts of non-isotropic dilations are also deeply considered. This is a jointed work with Dr. Huiju Wang.

(19) **Satoshi Masaki** (Hokkaido University, masaki@math.sci.hokudai.ac.jp)
Title: Global dynamics below first excited solitons for 3d cubic NLS with potential without radial symmetry.
Abstract: We consider the cubic nonlinear Schrodinger equation with a linear potential in 3d. The linear potential is chosen so that the corresponding Schrodinger operator possesses one simple negative eigenvalue. Under this setting, the nonlinear equation has stable ground states and unstable excited states, at least under small mass constraints. We consider the global dynamics of solutions with small mass and energy less than the first excited states. It turns out that such a solution scatters to a ground state for both time directions or blows up in finite time for both time directions. This result is an extension of Nakanishi '16, where the same result is established with radial symmetry. This is based on joint work with J. Murphy and J. Segata.

(20) **Shohei Nakamura** (Osaka University / University of Birmingham, srmkn@math.sci.osaka-u.ac.jp)

Title: A study of the periodic Zakharov system via decoupling theory.
Abstract: This talk is based on a joint work with Shinya Kinoshita (Tokyo institution of technology) and Akansha Sanwal (Universität Innsbruck).

In this talk we concern the local well-posedness of the periodic Zakharov system with low regularity on $\mathbb{T}^d$ with $d \geq 3$. The sharp regularity ensuring the local well-posedness is known for $d = 1$ due to Takaoka and $d = 2$ due to Kishimoto. For $d \geq 3$ the problem was addressed by Kishimoto and he obtained some partial progress. Our aim is to improve the Kishimotos local well-posedness result for $d \geq 3$ and moreover provides almost sharp result (as long as one utilizes the iteration argument) up to ε-loss in terms of the regularity for $d = 3$ and $d \geq 5$. To this end we introduce an idea from recent developments of the decoupling theory (Wolffs inequality). In fact, at the same time, the study of the periodic Zakharov system raises a question on some trilinear estimates involving free solutions of Schrödinger and wave equations on torus which can be regarded as some trilinear decoupling type estimates involving paraboloid and cone.

(21) **Jaehyeon Ryu** (Korea Institute of Advanced Study, jhryu@kias.re.kr)
Title: Almost everywhere convergence of Bochner-Riesz means for the twisted Laplacian.
Abstract: We study almost everywhere convergence of the Bochner-Riesz means associated with the twisted Laplacian. In the talk, we present the result stating that, for $2 \leq p \leq \infty$, the sharp summability index for the convergence is half of the index appearing in the result for the classical Bochner-Riesz mean. An interesting aspect of the proof is that it does not require the use of the A_p weight theory, unlike the case of the classical Bochner-Riesz mean. This work is joint work with Eunhee Jeong and Sanghyuk Lee.

(22) **Yoshihiro Sawano** (Chuo University, yoshihiro-sawano@celery.ocn.ne.jp)
Title: Atomic decomposition of a subspace of BMO.
Abstract: The goal of my talk is to introduce a new decomposition of a subspace of BMO. This is a continuation of what I have been doing for other function spaces. Around 1990, Frazier and Jawerth introduced the technique of obtaining non-smooth atoms from wavelet decomposition. This idea was revisited by Grafakos in his text book Modern Harnomic Analysis. This technique together with the reexamination of the atomic decomposition of Hardy spaces with variable exponents brought out a new technique to decompose functions in other spaces such as Triebel-Lizorkin-Morrey spaces. My talk reports an advancement in this direction.

(23) **Zhongwei Shen** (University of Kentucky, zshen2@uky.edu)
Title: Nodal and Critical Sets of Elliptic Equations in Homogenization.
Abstract: In this talk I will describe my recent work, joint with Fanghua

Lin, on the geometric properties of solutions of partial differential equations in the homogenization theory. We consider second-order elliptic equations with rapidly oscillating and periodic coefficients. We show that the (d-1)-dimensional Hausdorff measures of the nodal sets and the $(d-2)$-dimensional Hausdorff measures of the critical sets are bounded uniformly with respect to the period, provided that the doubling indices for the solutions are bounded. The proof uses the harmonic approximation successively. The key is to control accumulated errors by renormalization and rescaling.

(24) **Naoto Shida** (Nagoya University, naoto.shida.c3@math.nagoya-u.ac.jp)
Title: Boundedness of multilinear pseudo-differential operators with $S_{0,0}$ class symbols on Besov spaces.
Abstract: We consider multilinear pseudo-differential operators with symbols in the multilinear Hörmander class $S_{0,0}$. The boundedness of these operators on Lebesgue spaces has been established by Miyachi-Tomita (2013) in the bilinear case and by Kato-Miyachi-Tomita (2022) in the multilinear case. In this talk, we discuss the boundedness of these operators in the settings of Besov spaces.

(25) **Soichiro Suzuki** (Chuo University, soichiro.suzuki.m18020a@gmail.com)
Title: Energy decay estimates of the fractional Klein–Gordon equation with space-dependent damping.
Abstract: We consider the fractional Klein–Gordon equation on $\mathbb{R}^d$ with space-dependent damping: $u_{tt} + (-\Delta + 1)^{s/2}u + b(x)u_t = 0$. It is standard to show that the energy $E(t) = \|(u, u_t)\|^2_{H^{s/2}\times L^2}$ is non-increasing. In particular, it decays exponentially if $b(x)$ is a positive constant. Recently, Green (2020) and Green–Jaye–Mitkovski (2022) established a necessary and sufficient condition for exponential decay by using a kind of the uncertainty principle in Fourier analysis. Motivated by these results, we obtain a necessary and sufficient condition for logarithmic decay and a sufficient condition for polynomial decay. In this talk, I will give a brief overview of these results, particularly how the uncertainty principle is related to energy decay estimates. This is joint work with Kotaro Inami (Nagoya University).

(26) **Hitoshi Tanaka** (National University Corporation Tsukuba University of Technology, htanaka@k.tsukuba-tech.ac.jp)
Title: Fractional operators on weak Choquet spaces and Choquet-Morrey spaces with Hausdorff capacities.
Abstract: It is shown that fractional integral and maximal operators are bounded on weak Choquet spaces with Hausdorff capacity. We also investigate these operators on Choquet-Morrey spaces. These results are extentions of the previous works of Adams, Orobitg and Verdera, and Tang.
This is joint work with N. Hatano, R. Kawasumiand H. Saito.

(27) **Hanli Tang** (Beijing Normal University, hltang@bnu.edu.cn)
Title: Stability of Hardy-Littlewood-Sobolev inequalities with explicite lower bounds.
Abstract: We establish the stability for the Hardy-Littlewood-Sobolev inequalities with explicit lower bounds. By establishing the relation between the stability of HLS inequalities and the stability of fractional Sobolev inequalities, we also give the stability of the fractional Sobolev inequalities with lower bounds. This is a joint work with L.Chen and G.Lu.

(28) **Yutaka Terasawa** (Nagoya University, yutaka@math.nagoya-u.ac.jp)
Title: Liouville-type theorems for the Taylor–Couette flow of the stationary Navier–Stokes equations .
Abstract: We study the stationary Navier–Stokes equations in the region between two rotating concentric cylinders. We first prove that, under the small Reynolds number, if the fluid is axisymmetric and if its velocity is sufficiently small in the L^∞-norm, then it is necessarily a generalized Taylor-Couette flow. If, in addition, the associated pressure is bounded or periodic in the z-axis, then it coincides with the well-known canonical Taylor-Couette flow. Next, we give a certain bound of the Reynolds number and the L^∞-norm of the velocity which imply the fluid is axisymmetric. It is clarified that smallness of Reynolds number of the fluid in the two rotating concentric cylinders implies both axisymmetry and the exact form of the Taylor-Couette flow.

This talk is based on a joint work with Professor Hideo Kozono (Waseda Univ./Tohoku Univ.) and Professor Yuta Wakasugi (Hiroshima Univ.).

(29) **Naohito Tomita** (Osaka University, tomita@math.sci.osaka-u.ac.jp)
Title: The boundedness of some bilinear wave operators.
Abstract: In this talk, we consider bilinear Fourier multipliers related to wave operators, and bilinear Fourier integral operators are also discussed. Our result gives an improvement of Rodríguez-López, Rule, and Staubach which is a bilinear version of the classical theorem of Seeger, Sogge and Stein for Fourier integral operators. This is a joint work with Tomoya Kato and Akihiko Miyachi.

(30) **Yohei Tsutsui** (Kyoto University, ytsutsui@math.kyoto-u.ac.jp)
Title: Local solutions to the incompressible forced Navier-Stokes equations with smooth and large data.
Abstract: We construct a unique local solution to the incompressible Navier-Stokes equations on the whole space when initial data is smooth and large, and also the external force is continuous at initial time. Our result can be regarded as a counterpart of non-uniqueness of weak solutions by D. Albritton, E. Bru and M. Colombo (2022). Their external force is integrable near initial time.

(31) **Changhun Yang** (Chungbuk National University, chyang@chungbuk.ac.kr)
Title: The modified scattering for Dirac equations of scattering-critical nonlinearity.
Abstract: In this talk, we consider the Maxwell-Dirac system in 3 dimensions under zero magnetic field. We prove the global well-posedness and modified scattering for small solutions in the weighted Sobolev class. Imposing the Lorenz gauge condition, and taking the Dirac projection operator, it becomes a system of Dirac equations with Hartree type nonlinearity with a long range potential as $\frac{1}{|x|}$. We perform the weighted energy estimates. In this procedure, we have to deal with various resonance functions that stem from the Dirac projections. We use the spacetime resonance argument of Germain-Masmoudi-Shatah, as well as the spinorial null-structure. On the way, we recognize a long range interaction which is responsible for a logarithmic phase correction in the modified scattering statement.

(32) **Wen Yuan** (Bejing Normal University, wenyuan@bnu.edu.cn)
Title: Brezis-Van Schaftingen-Yung and Bourgain-Brezis-Mironescu Formulae in Ball Banach Function Spaces.
Abstract: Let X be a Ball Banach function space. In this talk, we first recall the Bourgain-Brezis-Mironescu and the Brezis-Van Schaftingen-Yung formulae related to of the Sobolev space $W^{1,1}(\mathbb{R}^n)$. Then, under some mild assumptions, and via a new method involving extrapolation, we establish the Brezis-Van Schaftingen-Yung formula and the Bourgain-Brezis-Mironescu formula in a more general setting of Ball Banach function space X. This generalization has a wide range of applications and, particularly, enables us to establish new fractional Sobolev and Gagliardo-Nirenberg inequalities in various function spaces, including Morrey spaces, mixed-norm Lebesgue spaces, variable Lebesgue spaces, weighted Lebesgue spaces, Orlicz spaces, and Orlicz-slice (generalized amalgam) spaces.

(33) **Junyong Zhang** (Beijing Institute of Technology, zhang_junyong@bit.edu.cn)
Title: Uniform resolvent estimates for critical magnetic Schrodinger operator in 2D.
Abstract: In this talk, we present the L^p-L^q type uniform resolvent estimates for 2D Schrodinger operator in scaling-critical magnetic fields, involving the Aharonov-Bohm model as a main example. As an application, we prove localization estimates for the eigenvalue of some non self-adjoint zero-order perturbation of the magnetic Hamiltonian. This is based on a joint work with L. Fanelli(Bilbao), J. Zheng (IAPCM)

(34) **Lu Zhang** (Shaanxi Normal University, luzhang_math@snnu.edu.cn)
Title: Almost sharp estimate for multi-parameter multi-linear Hörmander multipliers.
Abstract: In this talk, we investigate the Hörmander-type theorems for the

multi- linear and multi-parameter Fourier multipliers. When the multipliers are characterized by L^u- based Sobolev norms for $1 < u \leq 2$, our results on the smoothness assumptions are sharp in the multi-parameter and bilinear case. In the multi-parameter and multi-linear case, our results are almost sharp. This is a joint work with J. Chen, D. He, G. Lu and B. Park.

(35) **Jiuyi Zhu** (Louisiana State University, zhu@math.lsu.edu)
Title: Spectral inequality for schrodinger equations with power growth potentials.
Abstract: We obtain a sharp spectral inequality for Schrodinger equations with power growth potentials. This sharp spectral inequality depends on the radius and thickness of the sensor sets, and the growth rate of the potentials. The proof relies on quantitative global and local Carleman estimates to obtain quantitative three-ball inequalities. The work is joint with Jinping Zhuge.

(36) **Jinping Zhuge** (Chinese Academy of Sciences, jpzhuge@amss.ac.cn)
Title: Nodal sets of eigenfunctions in quasiconvex Lipschitz domains.
Abstract: Estimating the size of the nodal sets for the eigenfunctions of elliptic operators is a classical unique continuation problem, which has had several breakthroughs recently due to A. Logunovs work. In this talk, I will present our recent work on the estimate of nodal sets in quasiconvex Lipschitz domains which generalizes the corresponding result in C^1 domains by Logunov-Malinnikova-Nadirashvili-Nazarov (GAFA, 2021). The quasiconvex Lipschitz domains is a unified class of Lipschitz domains that contains both C^1 and convex domains. Particularly, our result is new and sharp for Laplace operator in convex domains. This is a joint work with Jiuyi Zhu.